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BRIEF COMMUNICATION

FLOW PATTERN TRANSITION IN ROUGH PIPES

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1. INTRODUCTION

Recently, a general model for prediction flow regime transition in horizontal and near horizontal gas–liquid flow was presented (Taitel & Dukler 1976*a*). Although the model was quite general it was limited to smooth pipes. The purpose of this note is to extend the aforementioned work to include the effect of roughness.

2. ANALYSIS

The analysis is based on the model presented in Taitel & Dukler (1976*a*); thus only modification needed to include roughness will be presented.

The general approach is to consider an equilibrium flow in a pipe, to calculate the equilibrium level in the pipe for given flow rates of gas and liquid and to examine whether this configuration is stable, and if not, to predict the mechanism of transition and the resulting flow pattern.

The starting point of the analysis is a momentum balance on each phase, assuming equilibrium stratified flow; thus for the liquid phase, we obtain

$$-A_L\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)-\tau_{WL}S_L+\tau_iS_i+\rho_LA_Lg\sin\alpha=0 \qquad [1]$$

while for the gas phase

$$-A_G\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)-\tau_{WG}S_G-\tau_iS_i+\rho_GA_Gg\sin\alpha=0 \qquad [2]$$

is valid.

In these equations A is the flow cross sectional area, τ_W is the stress at the wall, and S is the perimeter over which the stress acts. The subscript L refers to liquid, G to gas and i to the interface. α is the angle between the pipe axis and the horizontal, positive for down flow and g is the acceleration of gravity.

If the pressure gradient $\mathrm{d}P/\mathrm{d}x$ is eliminated, one gets a single equation for the liquid level in the pipe. The derivation is along the same lines as presented by Taitel & Dukler (1976*a*) and details are omitted. The final equation in a dimensionless form can be written as follows:

$$X^2\frac{f_L}{f_{LS}}\frac{\tilde{U}_L{}^2\tilde{S}_L}{\tilde{A}_L}-\frac{f_G}{f_{GS}}\frac{\tilde{U}_G{}^2S_G}{\tilde{A}_G}-\frac{f_i}{F_{GS}}U_G{}^2\left(\frac{\tilde{S}_i}{\tilde{A}_L}+\frac{\tilde{S}_i}{\tilde{A}_G}\right)-4Y=0. \qquad [3]$$

The dimensionless variables are designated by a tilde (˜). The perimeters S are normalized with respect to the diameter D, area is scaled by D^2, the liquid average velocity U_L is normalized by the superficial liquid velocity U_{LS}; likewise the gas velocity is scaled by U_{GS} (S

being a subscript to indicate superficial flow, namely, when only one phase is flowing) f_L is the friction-factor coefficient defined by

$$\tau_{WL} = f_L \frac{\rho_L U_L^2}{2} \qquad [4]$$

likewise

$$\tau_{WG} = f_G \frac{\rho_G U_G^2}{2} \qquad \tau_i = f_i \frac{\rho_G U_G^2}{2} \qquad [5]$$

while f_{LS}, f_{GS} are the coefficients that refer to the flow of a single phase, liquid or gas, respectively. X is Martinelli parameter defined by

$$X^2 = \frac{\frac{4}{D} f_{LS} \frac{\rho_L U_{LS}^2}{2}}{\frac{4}{D} f_{GS} \frac{\rho_G U_{GS}^2}{2}} = \frac{(\mathrm{d}P/\mathrm{d}x)_{LS}}{(\mathrm{d}P/\mathrm{d}x)_{GS}} \qquad [6]$$

and

$$Y = \frac{(\rho_L - \rho_G) g \sin\alpha}{(\mathrm{d}P/\mathrm{d}x)_{GS}}. \qquad [7]$$

Equation [3] has to be solved for the equilibrium liquid level $\tilde{h}_L = h_L/D$ in the pipe. Note that all variables with the superscript ~ depend on $\tilde{h}_L$; thus the solution of [3] depends on Martinelli parameter X, the inclination parameter Y and the ratios f_L/f_{LS}, f_G/f_{GS} and f_i/f_{GS}.

Fortunately it seems that in practice the above mentioned ratios are relatively weak functions of the dependent variables and fairly close to unity. As a result we expect the solution for $\tilde{h}_L$ to be fairly independent of these ratios.

Taitel & Dukler (1976*a*) considered smooth tubes where the friction factor can be correlated by the Blasius equation, namely by:

$$f_L = C_L \left(\frac{D_L U_L}{\nu_L}\right)^{-n} \qquad f_G = C_G \left(\frac{D_G U_G}{\nu_G}\right)^{-m} \qquad [8]$$

where C_L, C_G and n, m were taken as 16 and 1 for laminar flow respectively and 0.046 and 0.2 for turbulent flow. Note that D_L and D_G are the hydraulic diameters; thus $D_L = 4A_L/S_L$ for the liquid and $D_G = 4A_G/(S_G + S_i)$ for the gas. Furthermore, it was shown (Taitel & Dukler 1976*b*) that $f_i/f_G \simeq 1$ is a workable assumption; thus, in this case $f_L/f_{LS} = (\tilde{D}_L \tilde{U}_L)^{-n}$ while $f_G/f_{GS} = f_i/f_{GS} = (\tilde{D}_G \tilde{U}_G)^{-m}$. The solution has been found fairly the same for the four combinations of turbulent liquid–turbulent gas, turbulent liquid–laminar gas, laminar liquid–turbulent gas and laminar liquid and gas.

Considering now rough tubes, an expression for f valid for a wide range of Reynolds number Re is

$$\frac{1}{\sqrt{f}} = 3.48 - 4 \log_{10}\left(2\frac{e}{D} + \frac{9.35}{Re\sqrt{f}}\right) \qquad [9]$$

where e/D is the relative roughness.

Equation [9] is somewhat complex to use, since the friction factor f is given implicitly. If, however, we consider the limiting case of large Reynolds number, f is given explicitly and $f_L/f_{LS} = f_G/f_{GS} = 1$. A solution for this case (for $f_i/f_G \simeq 1$) is shown in figure 1 by the broken line as it is compared with the solution of turbulent-turbulent flow in smooth tubes. As expected, it demonstrates again that indeed the solution for the liquid level h_L/D is fairly unique function of X and Y.

Considering now transition boundaries among five basic flow patterns: stratified smooth, stratified wavy, intermittent (slug and plug), annular and dispressed bubble as outlined in Taitel & Dukler (1976*a*), one can observe that conditions of transition are the same for all transition lines except for that of the transition between the intermittent to the dispersed bubble flow pattern. Following the derivation for the condition that leads to such transition in the general case yields

$$T^2 \geq \frac{8\tilde{A}_G}{\tilde{S}_i \tilde{U}_L^2 f_L/f_{LS}} \qquad [10]$$

where

$$T = \left[\frac{(\mathrm{d}P/\mathrm{d}x)_{LS}}{(\rho_L - \rho_G) g \cos \alpha}\right]^{1/2}. \qquad [11]$$

For rough tubes and large Reynolds number $f_L/f_{LS} = 1$. As can be seen, f_L/f_{LS} is very close to unity also for smooth tubes (especially since this transition occurs for relatively high liquid flow rate, where the flow of liquid is close to the flow of a single phase).

The consequence of this analysis leads to the conclusion that the prediction for flow pattern transition for smooth tubes in terms of the five dimensionless parameters X, Y, F, T and K (Taitel & Dukler 1976*a*) is also valid for rough pipes provided that $(\mathrm{d}P/\mathrm{d}x)_{LS}$ and $(\mathrm{d}P/\mathrm{d}x)_{GS}$ is calculated for rough pipes. Likewise our generalized map presented for horizontal pipes where all transition lines are mapped on a single two dimensional plot is also applicable for rough pipes.

To further demonstrate the effect of roughness, figure 2 shows a map where the superficial velocities of the liquid U_{LS} and gas U_{GS} are used as coordinates. In this figure the transition lines for smooth and rough pipes are considered. For rough pipes, [9] was used (not limited to large Re). The results show that the only noticeable difference is the intermittent-dispersed bubble transition.

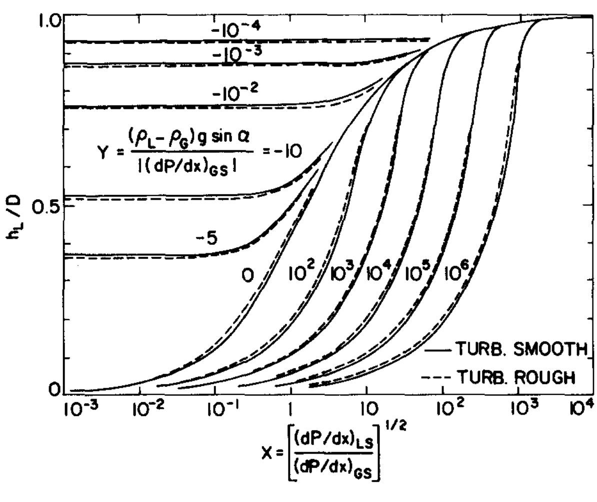

Figure 1. Equilibrium liquid level for stratified flow (turbulent smooth and turbulent rough).

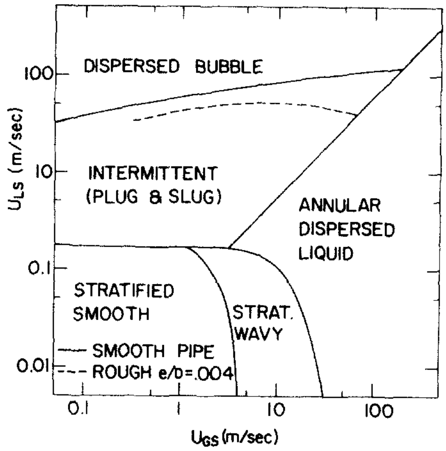

Figure 2. Effect of roughness on transition boundaries, water–air, 25°C, 1 atm, 5 cm diam., horizontal.

This is actually what we would expect on physical grounds. Since roughness affects both liquid and gas pressure drop, we expect little change in equilibrium liquid level in the pipe (or liquid hold up). As a result most transitions are also insensitive to roughness. This excludes the transition between intermittent and dispersed bubble. As pointed out by the theory, this transition occurs when turbulent pressure fluctuations exceed buoyancy forces. For rough pipes, enhancement of turbulence leads to transition to dispersed bubble at lower liquid flow rate compared to the case of smooth tubes.

For inclined pipes, however, we expect all transitions to be affected by roughness. This is because roughness increases frictional pressure drop while the gravitational one remains unaffected by roughness. Indeed, figure 3 shows that roughness causes early transition to intermittent or annular flow patterns for increasing liquid and gas flow rates.

Although no generalized map is provided for transition boundaries with inclination, since this requires mapping of transition lines with respect to three dimensionless variable, it is easy to use the equations for the transitions directly.

SUMMARY AND CONCLUSIONS

The method of predicting the flow pattern (Taitel & Dukler 1976*a*) has been extended to rough pipes. It has been demonstrated that the generalized map as presented in the aforemen-

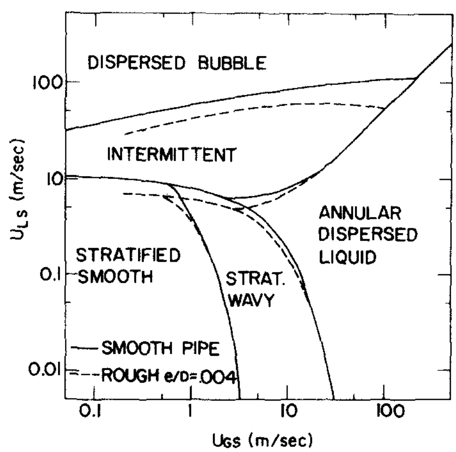

Figure 3. Effect of roughness on transition boundaries. Water–air, 25°C, 1 atm. 5 cm diam., downflow 1°.

tioned method is directly applicable provided that $(dP/dx)_{LS}$ and $(dP/dx)_{GS}$ that appear in the Martinelli parameter X and the dimensionless variable Y [7] are calculated for rough pipes.

Effect of roughness has been shown to be negligible in the horizontal case for all transition boundaries except for the intermittent-dispersed bubble transition. For inclined pipes the effect of roughness is shown to affect all transition boundaries.

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